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Sting-free measurements of sphere drag in laminar flow

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An aerodynamic investigation was conducted to determine the laminar-flow drag coefficient of spheres of various sizes in a subsonic wind tunnel. The tests were conducted using the M.I.T.–N.A.S.A. prototype magnetic-balance system. By measuring the drag of different sized spheres without model support interference the tunnel wall effect can be deduced. The present results indicate that the classical wind tunnel correction does not completely account for the effects of model size and wall interference. That is, the corrected drag coefficient data for the different sphere sizes differ among themselves in the region of Reynolds number overlap.

A comparison of the present sphere drag results with those of numerous other investigations including free-flight and ballistic-range data is given. The drag coefficients presented here are slightly lower than those of other workers for Reynolds numbers ranging from 20 000 to 150 000, but fall between the limits of experimental scatter for Reynolds numbers from 150 000 to 260 000.

An analysis of the estimated error in the present data indicates the primary source to be measurement of the wind tunnel parameters rather than errors resulting from the balance system.

1. Introduction

The flow field characteristics and drag of spheres has been a classical problem requiring both theoretical and experimental work for many years (Hoerner 1965). The pressures calculated from potential theory are in fairly close agreement with experimental results over the forward portion of the sphere. However, the effects of viscosity dominate the flow over the rear portion of the sphere. The location of the boundary-layer separation point seems to be in doubt and may be the source of the disagreement between the theoretical drag computations and the sphere drag obtained by experiment. In addition, results of different experimental investigations over the years have shown the measurement of sphere drag to be influenced by experimental conditions such as model supports, flow turbulence and the wind tunnel wall interference. The effect of flow turbulence on the measured sphere drag at transition Reynolds numbers is used as a criterion for determining the turbulence level of a wind tunnel (Pankhurst & Holder 1952). In recent years the measurement of sphere drag has had application to spherically shaped balloons used to probe atmospheric density (see *N.A.S.A.* SP-219, 1969).

2. Apparatus

The magnetic balance used in these tests is described in detail† by Stephens (1969). The forces on the model are computed from the measured magnet coil currents required to balance the aerodynamic and gravity loads. The measured magnet currents, tunnel conditions and model position data are processed by a computer program which reduces the data to aerodynamic coefficient form. The data reduction techniques developed for this balance are discussed in detail by Gilliam (1969).

The subsonic wind tunnel used in these tests is an open-circuit closed-jet tunnel with intake open to the test room. A continuous variation in velocity from 0 to 550 ft/s can be obtained at the test section. This corresponds to a maximum dynamic pressure of 2·5 psi and free-stream Reynolds number of $3{\cdot}5 \times 10^6$/ft. The test section is octagonal, with an inside dimension of $6\frac{1}{2}$ in. The tunnel was designed to produce a low turbulence flow in the test section. A summary of the properties of the tunnel flow is presented in Judd, Vlajinac & Covert (1971)‡. An indication of the low tunnel turbulence is the fact that the present sphere drag measurement at a Reynolds number of 310000 do not show natural transition.

The models used in these tests were spheres with nominal diameters of 0·75, 1·00 and 1·500 in. They were machined from Armco magnetic ingot iron round stock which had been annealed at 700 °F prior to machining. The model surface had an r.m.s. finish of 16 μin. The maximum variation in sphericity was $\pm$ 0·001 in.

3. Description of tests

Tests on the three spheres, with nominal diameters of 0·75, 1·00 and 1·50 in., were conducted to determine the drag over a Reynolds number range from 10^4 to $2{\cdot}7 \times 10^5$. Overlap in the drag data for all three sizes was obtained for Reynolds numbers from $0{\cdot}59 \times 10^5$ to $0{\cdot}87 \times 10^5$. Additional overlap was obtained for pairs of the sphere sizes. The maximum Mach number in these tests was 0·30. Under these circumstances the maximum density change over the sphere is about 10 % so the flow is essentially incompressible.

The drag, lift, and side force on the sphere were obtained by measuring the magnet coil currents required to balance the model weight (gravity) and aerodynamic loads. The magnet currents were measured with an integrating digital voltmeter. The integration (averaging) period for each current measurement was 10 s. The 10 s sampling attenuates the effects of ripple and noise in the magnet currents, thus providing an accurate average of the coil current from which the steady-state loads on the model can be obtained.

† The suspension of a spherical configuration required a considerable increase in the magnetizing coil power owing to the large demagnetizing factor for a sphere compared with that for models with slenderness ratios greater than one (see Bozorth 1952, p. 845). An additional power supply has been connected to the magnetizing coils.

‡ The maximum turbulence level during these tests was less than 0·25 % of the mean-square fluctuating velocity (mean velocity). At a Reynolds number of $1{\cdot}0 \times 10^6$ the measured turbulence drops to 0·07 %.

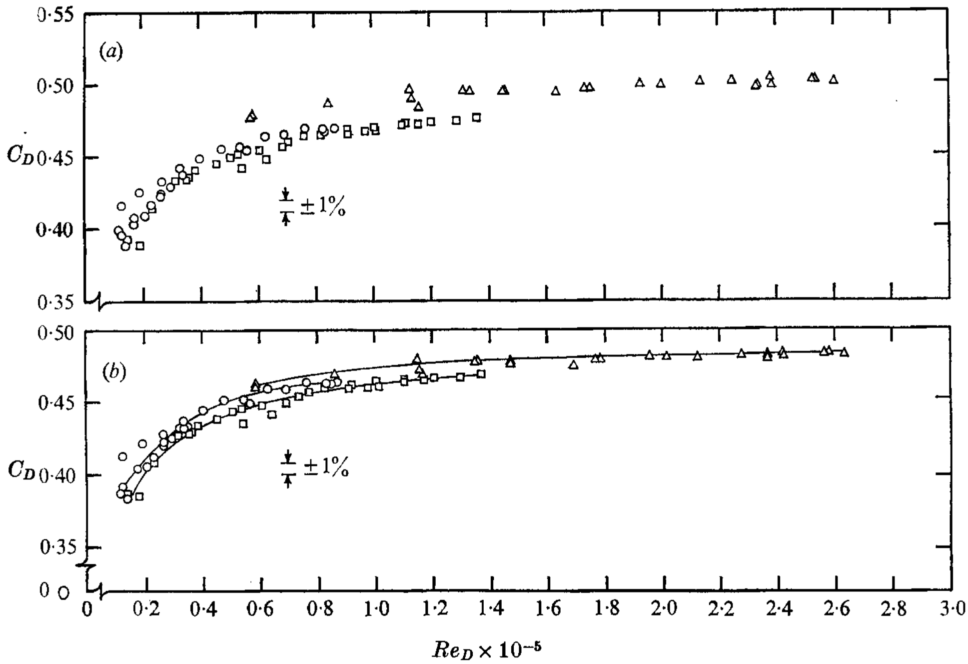

FIGURE 1. Corrected and uncorrected drag coefficient C_D versus Reynolds number Re_D. ○, $D = 0{\cdot}75$ in., □, $D = 1{\cdot}00$ in.; △, $D = 1{\cdot}50$ in., where D is the diameter of the sphere. (*a*) Uncorrected value. (*b*) Corrected value.

The magnet currents were also measured with zero wind. This provided the tare currents which are required in the data reduction. A considerable simplification in both data reduction and calibration is obtained in the case of a sphere because, from symmetry, the pitch and yaw positions are not required nor is moment control in the balance. Thus symmetry eliminates interaction terms in the calibration.

The relation between the drag load on the model and the drag coil current was obtained by applying known drag loads to the suspended model in the direction of the wind tunnel axis. For this the linearity between applied drag load and drag magnet current was within 0·3 % over the entire range of aerodynamic loads. The nonlinear effect was accounted for in the data reduction program (Gilliam 1969), thus reducing the maximum balance error in data reduction to 0·1 %.

4. Test results and discussion

The drag coefficient data, Reynolds number and Mach number for the three spheres are given in table 1. These data have been corrected for solid blockage and wake blocking using the methods described in Pankhurst & Holder (1952). Both the corrected and uncorrected data are shown in figure 1 to illustrate the effect of blockage corrections on the reduced data. For all three spheres the effect of blockage corrections is to reduce the drag coefficient at a given Reynolds number. However, it can be seen from the plot of the corrected data that the

0·75 in. diameter sphere

$Re_D \times 10^{-5}$	M	C_D
0·1136	0·0267	0·3882
0·1254	0·0296	0·4142
0·1268	0·0298	0·3935
0·1401	0·0392	0·3837
0·1684	0·0396	0·4012
0·1741	0·0410	0·4055
0·1936	0·0457	0·4228
0·2084	0·0490	0·4072
0·2302	0·0542	0·4140
0·2635	0·0620	0·4207
0·2658	0·0627	0·4244
0·2663	0·0629	0·4297
0·2988	0·0703	0·4266
0·3331	0·0787	0·4383
0·3407	0·0802	0·4340
0·3439	0·0808	0·4350
0·4050	0·0957	0·4460
0·4763	0·1125	0·4530
0·5457	0·1289	0·4538
0·5672	0·1338	0·4516
0·6292	0·1486	0·4609
0·6934	0·1638	0·4615
0·7676	0·1813	0·4659
0·8343	0·1972	0·4654
0·8502	0·2006	0·4649
0·8780	0·2074	0·4666

1·00 in. diameter sphere

$Re_D \times 10^{-5}$	M	C_D
0·1423	0·0253	0·3884
0·1834	0·0327	0·3861
0·2312	0·0411	0·4106
0·3199	0·0568	0·4293
0·3535	0·0631	0·4300
0·3622	0·0645	0·4334
0·3889	0·0691	0·4358
0·5060	0·0899	0·4453
0·5379	0·0958	0·4463
0·5486	0·0979	0·4370
0·6081	0·1080	0·4499
0·6422	0·1146	0·4423
0·6973	0·1238	0·4514
0·7345	0·1310	0·4552
0·7751	0·1381	0·4588
0·8270	0·1475	0·4609
0·9211	0·1643	0·4608
0·9245	0·1649	0·4640
0·9794	0·1745	0·4621
1·017	0·1815	0·4647
1·023	0·1825	0·4639
1·111	0·1983	0·4661
1·116	0·1991	0·4679
1·173	0·2089	0·4670
1·208	0·2156	0·4680
1·302	0·2323	0·4694
1·372	0·2445	0·4712

TABLE 1

1·50 in. diameter sphere

$Re_D \times 10^{-5}$	M	C_D
0·5856	0·0680	0·4615
0·5870	0·0681	0·4646
0·8619	0·1010	0·4718
1·148	0·1332	0·4819
1·159	0·1345	0·4746
1·173	0·1361	0·4690
1·348	0·1565	0·4798
1·359	0·1578	0·4794
1·475	0·1712	0·4792
1·477	0·1714	0·4795
1·669	0·1938	0·4777
1·769	0·2054	0·4813
1·786	0·2073	0·4819
1·961	0·2276	0·4837
2·042	0·2370	0·4830
2·132	0·2481	0·4835
2·285	0·2659	0·4854
2·371	0·2752	0·4837
2·371	0·2752	0·4817
2·424	0·2822	0·4877
2·425	0·2823	0·4830
2·570	0·2991	0·4863
2·585	0·3009	0·4864
2·648	0·3081	0·4849

TABLE 1 (*cont.*)

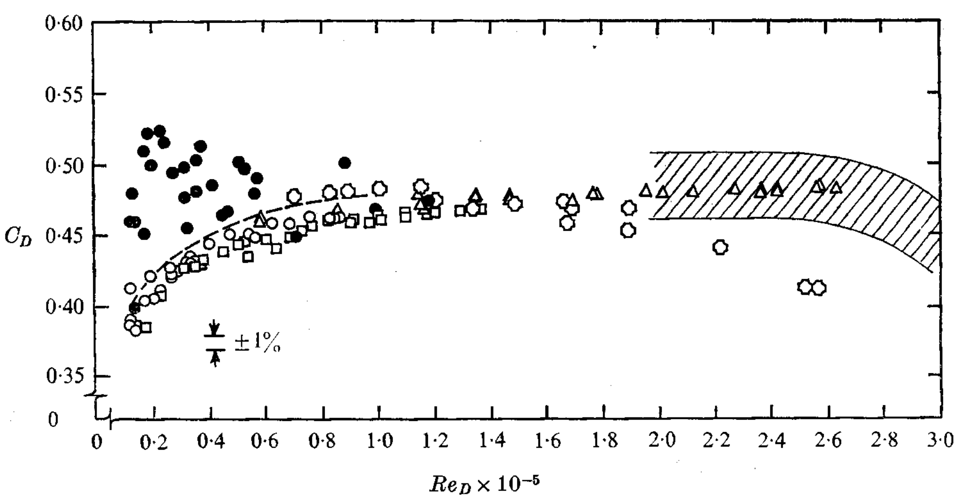

FIGURE 2. Drag coefficient versus Reynolds number. Present data: ○, D = 0·75 in.; □, D = 1·00 in., △, D = 1·50 in. ●, Roos & Willmarth (1971); ⬡, Hoerner (1935); – – –, ballistic range, Bailey & Hiatt (1971) and Goin & Lawrence (1968); /////, free flight, Millikan & Klein (1933).

data are not completely reduced to a single line. The scatter in the data for Reynolds numbers below $0{\cdot}4 \times 10^5$ is primarily due to errors in the wind tunnel dynamic pressure measurements. These errors are discussed in the appendix.

These results imply that the theoretical value of drag coefficient variation due to blockage, while qualitatively correct, is of limited accuracy. (The present collapse is within 1·5 %.) This is in agreement with the previous results obtained by Judd *et al.* 1971.

A comparison of the present data with the results of other tests (Roos & Willmarth 1971; Bailey & Hiatt 1971; Goin & Lawrence 1968; Hoerner 1935; Millikan & Klein 1933) is shown in figure 2. The present data, which are consistently lower, agree to within 1·5 % with ballistic-range data. The data for the 1·5 in. sphere are in excellent agreement with free-flight (Millikan & Klein 1933) data at the higher Reynolds numbers tested.

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Appendix. Error analyses

In estimating the overall accuracy of the force data several sources of error were considered. The accuracy with which each variable could be determined was the following:

$$\Delta I_x, \Delta I_D = \pm 0{\cdot}01\,\text{mV for magnet current measurements,}$$
$$\Delta S = \pm 2{\cdot}5 \times 10^{-7}\,\text{in.}^2 \text{ for the model area } S,$$
$$\Delta q = \pm 0{\cdot}0025\,\text{in. manometer fluid for the pressure } q,$$

where I_x is the magnetizing current and I_D is the drag current. The estimated error ΔC in the magnetic force coefficient for data reduction based on measurement of the magnet currents and calibration weights (W) is given by

$$\begin{aligned}\frac{\Delta C}{C} &= \left[\left(\frac{\Delta I_x}{I_x}\right)^2 + \left(\frac{\Delta I_D}{I_D}\right)^2 + \left(\frac{\Delta W}{W}\right)^2\right]^{\frac{1}{2}} \\ &= \left[\left(\frac{0{\cdot}01}{40}\right)^2 + \left(\frac{0{\cdot}01}{40}\right)^2 + \left(\frac{0{\cdot}003}{3}\right)^2\right]^{\frac{1}{2}} \\ &= 1{\cdot}06 \times 10^{-3}. \end{aligned} \tag{A 1}$$

The error ΔC_D incurred in the reduced range coefficient is given, on an equiprobability basis, by

$$\frac{\Delta C_D}{C_D} = \left[\left(\frac{\Delta C}{C}\right)^2 + 2\left(\frac{\Delta I_x}{I_x}\right)^2 + 2\left(\frac{\Delta I_D}{I_D}\right)^2 + \left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta q}{q}\right)^2\right]^{\frac{1}{2}}. \tag{A 2}$$

Since the magnetizing current I_x is approximately constant in all the tests, the value of the drag current I_D will vary directly with the tunnel pressure reading q

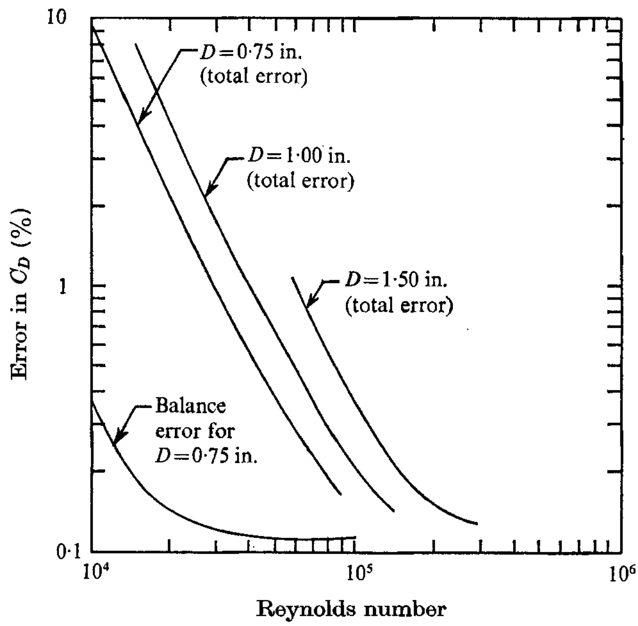

FIGURE 3. Estimated error in C_D versus Reynolds number.

(Gilliam 1969) if the drag coefficient is constant, which it essentially is. Equation (A 2) can be modified by assuming that

$$I_D \simeq Kq$$

and

$$\frac{\Delta C_D}{C_D} = \left[\left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta S}{S}\right)^2 + 2\left(\frac{\Delta I_x}{I_x}\right)^2 + 2\left(\frac{\Delta I_D}{Kq}\right)^2 + \left(\frac{\Delta q}{q}\right)^2\right]^{\frac{1}{2}}$$
$$= \left[\left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta S}{S}\right)^2 + 2\left(\frac{\Delta I_x}{I_x}\right)^2 + \frac{1}{q^2}\left(\frac{2\Delta I_D^2}{K^2} + (\Delta q)^2\right)\right]^{\frac{1}{2}},$$

using the value of K measured for the 0·75 in. sphere, namely 155, and substituting the values for the remaining variables given above. The result is

$$\Delta C_D/C_D = 10^{-3}[1·13 + 0·0006 + 0·13 + (1/q^2)(0·116 + 6·25)]^{\frac{1}{2}}$$
$$= 10^{-3}[1·2606 + (1/q^2)(0·116 + 6·25)]^{\frac{1}{2}}, \quad \text{(A 3)}$$

where q is measured in inches of manometer fluid. By examining the terms in round brackets in (A 3) one can see that the primary source of error shifts from the tunnel pressure measurement to the measurement of force by the balance as the tunnel dynamic pressure is increased.

The estimated error in the coefficient as a function of Reynolds number for the three sphere sizes tested is shown in figure 3. Also shown in figure 3 is the estimated error due to the balance parameters for the 0·75 in. diameter sphere. Comparison of the error in C_D due to the magnetic balance with the total error indicates that a substantially larger portion is produced by error in the wind tunnel parameters and suggests an improvement in future tests. The analysis of the present data error is seen to be consistent with the scatter of the data shown in figure 1.

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